SCHOOL: _SOLUTION_

TEAM: _____

RELAY: __1__

1. __4__
2. __8__
3. __80__
4. __5__

SCHOOL: _SOLUTION_

TEAM: _____

RELAY: __2__

1. __-1__
2. __7__
3. __2__
4. __-3__

SCHOOL: _SOLUTION_

TEAM: _____

RELAY: __3__

1. __3.5__
2. __1272__
3. __318__
4. __11__ ⟸ must be simplified

SCHOOL: _____

TEAM: _____

RELAY: _____

1. _____
2. _____
3. _____
4. _____

Sixth Annual Upper Peninsula
High School Math Challenge
Northern Michigan University (Marquette Co, MI)
Saturday 14 March 2015

RELAY 1   **PLAYER 1**

Category: ANALYTIC GEOMETRY

Find the y-coordinate of the point in the first quadrant where the line $x + y = 5$ intersects the parabola $y = x^2 + 2x + 1$.

Pass your answer to Player 2.

$$y = 5 - x$$

$$y = x^2 + 2x + 1$$

$$x^2 + 2x + 1 = 5 - x$$

$$x^2 + 3x - 4 = 0$$

$$(x + 4)(x - 1) = 0$$

$$x = 1 \quad \cancel{x = -4} \Leftarrow \text{First Quadrant}$$

$$y = 5 - x = 4$$

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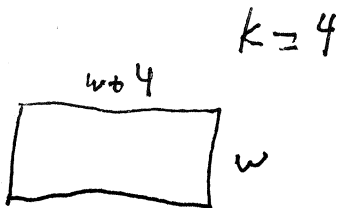## Northern Michigan University (Marquette Co, MI)
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RELAY 1   **PLAYER 2**

Category: PERIMETER AND AREA

Let k be the number you receive from Player 1. If the length of a rectangle is k cm more than its width and the area of the rectangle is 96 cm^2, find the number of centimeters in the rectangle's width.

Pass your answer to Player 3.

$k = 4$

$w + 4$

w

$w^2 + 4w = 96$

$w^2 + 4w - 96 = 0$

$(w + 12)(w - 8) = 0$

$w = 8 \quad w = \cancel{-12}$

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RELAY 1   **PLAYER 3**

Category: DIGIT SUMS

Let k be the number you receive from Player 2. Find the largest two-digit number whose digits sum to k.

Pass your answer to Player 4.

$$k = 8$$

MUST BE 80

RELAY 1   **PLAYER 4**

Category: PERFECT SQUARES

Let k be the number you receive from Player 3. Find the smallest positive integer x such that kx is a perfect square.

Run your answer to the front.

$80 = 2^4 \cdot 5$

multiplying by 5 gives $2^4 \cdot 5^2$

or a perfect square

RELAY 2   **PLAYER 1**

Category: EXPONENTIAL EQUATIONS

Find x if $\frac{4}{2^x} = 8$.

.

Pass your answer to Player 2.

$$\frac{2^2}{2^x} = 2^3$$

$$2^x = 2^2/2^3 = 2^{-1}$$

$$x = -1$$

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RELAY 2   **PLAYER 2**

Category: POLYNOMIAL ARITHMETIC

Let x be the number you receive from Player 1. Find the value of $(x + 2)(x^2 - 2x + 4)$.

Pass your answer to Player 3.

$x = -1$

Note that $(x+2)(x^2 - 2x + 4) =$
$$x^3 + 8$$

$(-1)^3 + 8 = 7$

RELAY 2   **PLAYER 3**

Category: INTEGER ARITHMETIC

Let n be the number you receive from Player 2. Find the larger of the two consecutive integers whose sum is equal to n and multiply that integer by one-half.

Pass your answer to Player 4.

$$n = 7$$

the 2 integers are 3 and 4

Half of that is 2.

RELAY 2　　　　　　　　　　　　　　　　　　　　　　**PLAYER 4**

Category: **SIMULTANEOUS EQUATIONS**

Let k be the answer you receive from Player 3. Find the x-value in the solution of the equations:

$6x + 4y = k.$
$y - x = 4k$

Run your answer to the front.

$$k = 2$$

$6x + 4y = 2$ $y - x = 8$
$-6x + 6y = 48$ $5 - x = 8$
_____ $x = -3$
$\quad 10y = 50$
$\quad\quad y = 5$

RELAY 3   **PLAYER 1**

Category: ALGEBRA

Two more than three times the difference between four and n is seven less than three times n. Find n.

Pass your answer to Player 2.

$$2 + 3(4-n) = 3n - 7$$

$$2 + 12 - 3n = 3n - 7$$

$$14 = 6n - 7$$

$$21 = 6n$$

$$n = \frac{21}{6} = \frac{7}{2} = 3.5$$

RELAY 3   **PLAYER 2**

Category: **VOLUME AND SURFACE AREA**

The number you will receive from Player 1 is the height, in **feet**, of a rectangular prism. If the length of the prism is eight **inches** and the width is three **feet** less than the height, find the number of **square inches** in the surface area of this prism.

Pass your answer to Player 3.

$$h = 3.5 \text{ ft} = 42 \text{ in}$$
$$\ell = 8 \text{ in}$$
$$w = 6 \text{ in}$$

$$SA = 2(42 \cdot 8 + 8 \cdot 6 + 42 \cdot 6$$
$$= 2(336 + 48 + 252)$$
$$= 2(636)$$
$$= 1272$$

RELAY 3   **PLAYER 3**

Category: PROBABILITY

The number you will receive from Player 3 is the number of quarters I have.

One quarter out of every one hundred six quarters is a two-headed quarter.

There are three times as many two-tailed quarters as two-headed quarters.

All of the other quarters are regular, fair quarters.

I flip each quarter, fair and unfair, *twice* in a row.

What is the average number of quarters on which I received heads both times I flipped it?

Pass your answer to Player 4.

I have q quarters.

$\dfrac{q}{106}$ are 2-headed

$\dfrac{3q}{106}$ are 2-tailed

regular quarters are $q - \dfrac{4q}{106} = q - \dfrac{2q}{53} = \dfrac{51q}{53}$

2-headed quarters <u>WILL</u> come up heads twice, $\dfrac{q}{106}$

2-tailed quarters <u>WILL NOT</u> come up heads twice $\quad 0$

regular quarters will come up heads twice $\frac{1}{4}$ of the time

$$\dfrac{51q}{53} \cdot \dfrac{1}{4} = \dfrac{51q}{212}$$

Total is $\dfrac{q}{106} + \dfrac{51q}{212} = \dfrac{2q}{212} + \dfrac{51q}{212} = \dfrac{53q}{212} = \dfrac{q}{4}$

Since q is 1272

$\dfrac{q}{4}$ is 318

RELAY 3   **PLAYER 4**

Category: QUADRATIC EQUATIONS

The number you receive from Player 3 is the constant, c, in the equation below. Find the larger of the two roots of that equation, in simplified form.

$$8x^2 - 2x - 628 = c$$

Run your answer to the front.

$c = 318$

$8x^2 - 2x - 628 = 318$

$8x^2 - 2x - 946 = 0$

$4x^2 - x - 473 = 0$

$(4x + 43)(x - 11) = 0$

$x = -\dfrac{43}{4} \qquad x = 11$